**Graph Review**

**Network = Graph** : A representation of connections among a set of objects. A graph is a set of nodes (vertices) joined by a set of lines or arrows (edge).

**Theory**

**The Seven Bridges of Königsberg** (the problem that is at the origin of graph theory) was posed by [Leonhard Euler](http://en.wikipedia.org/wiki/Leonhard_Euler) in 1735 (also prefigured the idea of topology)

The citizens of Königsberg supposedly walked about on Sundays trying to find a route that crosses each bridge Königsberg of exactly once, and return to the starting point.

**Network Types:**  
1. **Directed Network** – edges have direction

2. **Weighted network**: a network where edges are assigned a (typical numerical) weight.

3. **Multigraph:** A network where multiple edges (parallel edges) can connect the same nodes

**Edge Weight**

* **Distance** in a road network
* **Strength** of Connection in a personal network
* **Likelihood** of interaction in a biological network

**Path Analytics**

* **Walk:** an alternating sequence f vertices and edges over a graph.
  + H->F->G->C->F->E->B
  + Edges and nodes can be repeated
* **Path:** A walk with no repeating node **except possibly** for the first and last
  + J->G->C->F->E->B
* **Constraining a walk**
  + **Cycle**: a path of length n >= 3 whose start and end vertices are the same J->G->C->F->J
  + **Acyclic**: A walk with no cycle
* **Trail**: no edge can be repeated
* **Reachability**: node u is reachable from node v if there is a path from v to u

**Graph Attributes**

* **Graph Diameter:** the length of the **“longest shortest path”** between any two vertices (u,v). In other words, a graph's diameter is the largest number of vertices which must be traversed in order to travel from one vertex to another vertex.
* **Connectedness**: A graph is connected if it contains a directed path from u to v or a directed path from v to u for every vertex-pair (u,v)
* **Network Robustness:** If node v is reachable from node u originally, it should remain reachable even if the network is “attacked”
  + Attacked: Node/edge removed
* For a vertex, the number of head ends adjacent to a vertex is called the ***indegree*** of the vertex and the number of tail ends adjacent to a vertex is its ***outdegree***

**Centralities:**

* **Degree Centrality:** The number of edges connected to a node
  + exposure to the network
  + opportunity to directly influence.
* **Closeness Centrality:** The average of the shortest distance to all other nodes in the graph
  + estimates time to hear info
  + indirect influence
  + point of rapid diffusion
* **Betweenness Centrality:** Extent to which a particular node lies on the shortest path between other nodes
  + informal power
  + gate keeping, brokering controls flow of info
  + liaison between sub-components.
* **Eigenvalue Centrality:** A measure of the extend to which a node is connected to influential other nodes
  + connected to influential nodes of high degree, “not what you know but who you know”

***A clustering coefficient is a measure of the degree to which nodes in a graph tend to cluster together.***